Beyond Fresh Update: Packet Management for Real-Time Feedback Control

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***Abstract*—In real-time feedback control systems, the freshness of the packet is crucial to control performance, where packet management is vital to keep data fresh. Recently, *age of information* (AOI) has been used to measure the freshness of the update information, where minimizing AOI becomes popular in system designs. In this paper, we find that minimizing AOI is not always equivalent to maximizing the control performance. In particular, we define a metric, called the *age of stale information* (AOSI), to link the instability of the control system to AOI. By minimizing AOSI, we can maximize the control performance, and also reduce the communication cost.**

I. Introduction

In recent years, the proliferation of sensing, computing, and communication is revolutionizing the way that real-time control systems interact with the physical environment, e.g., temperature monitoring in air-conditioning systems and vehicle tracking in smart transportation systems [1][2]. In these feedback control systems, physical environment information should be timely sampled and transmitted to controllers, which effectively improves the control performance. Therefore, how to keep the information fresh is critical to real-time control systems.

In general, a metric, called the *age of information* (AOI), has been used to describe the freshness of information [3]. It is defined as

$$\Delta(t) = t - U(t), \tag{1}$$

where t is the current time instant, and $U(t)$ is the timestamp when the current information was generated, i.e., AOI is the time elapsed since the freshest packet was generated. By minimizing AOI, many packet update polices have been proposed to keep information fresh [4]-[12].

A typical information update process is shown in Fig. 1, where the information generated by the sensor is transmitted to the data center via a communication network with random time delays. For simplicity, a common assumption is that only one packet can be served at the same time [4]-[8]. In 2017, Yin Sun proposed a packet update policy, called the $\epsilon - wait$, to minimize AOI, where the packet needs to wait for a certain time before its transmission. This is because a packet is useful only if it carries some new information compared to the previous packet with a certain time gap. As a result, the data center would be able to obtain the fresh information.

However, if the information in data center is used to generate control commands and conduct physic actions though actuators, it becomes an open question on how to design the packet management method.

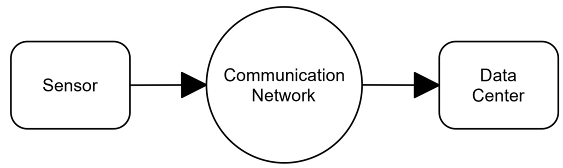

Fig. 1. A typical information update process.

In this paper, we answer the above question by investigating the relationship between control performance and AOI. In particular, we define a metric, called the *age of stale information* (AOSI), to link the instability of the control system to AOI. Based on this, we propose a new packet management method to minimize AOSI, where the control performance can be maximized.

The rest of this paper is organized as follows: In Section II, we provide the system model. Then, we discuss the relationship between control performance and AOI. In Section III, we propose a novel metric AOSI and then formulate the optimization problem. In Section IV, a new packet update policy, called the $\alpha - wait$, is proposed to solve the optimization problem. In Section V, simulation results are provided. Finally, Section VI concludes the paper.

II. System Model

A. Feedback Control System

An example[1] of a motion-tracking system is shown in Fig. 2 (a), where a human is trying to wirelessly control a robotic arm. Firstly, a Microsoft kinect camera captures the human's arm and obtain the position. Then, the position information is transmitted to the controller via a network. Finally, the controller control the robotic arm to reproduce the human's movement.

This is a typical networked feedback control system, which can be modeled as Fig. 2 (b). In the control loop, the sensor first samples the plant output $y(t)$ and the reference $r(t)$. The sensor generates a packet consist of the tracking error information $e(t) = r(t) - y(t)$. Then the packet is transmitted

[1]The detailed prototype demonstration video can be found at https://www.youtube.com/watch?v=zxDhQEcu4Vk&feature=youtu.be.

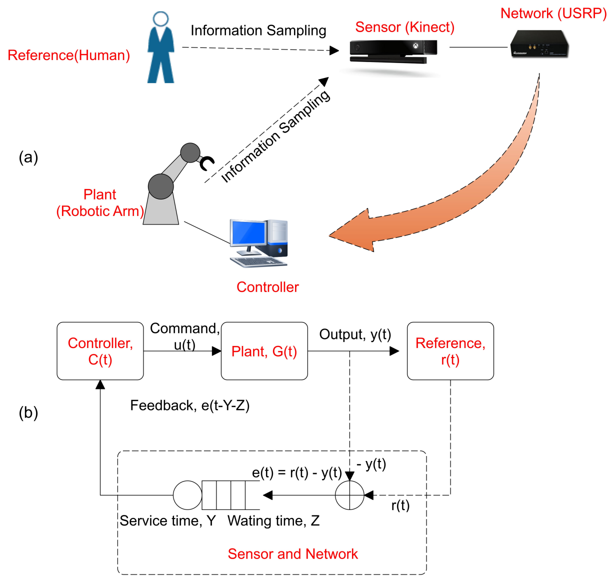

Fig. 2. A typical networked feedback control system.

to the controller via a network. After receiving the error information $e(t)$, the controller calculates a command $u(t)$ based on a specific control law $C(t)$. Then, the command $u(t)$ is going to be executed at the plant $G(t)$. Finally, we obtain the system output $y(t)$ which is a function of $G(t)$ and $u(t)$. Let $E(s)$, $G(s)$, $C(s)$, and $R(s)$ be the Laplace transforms of $e(t)$, $g(t)$, $c(t)$, and $r(t)$, the above control process can be expressed by

$$\big(R(s)-Y(s)\big)C(s)G(s)=Y(s). \tag{2}$$

By considering the control process with stale information, the control process (2) can be rewritten as

$$\big(R(s)-Y(s)\big)e^{-s\Delta(t)}C(s)G(s)=Y(s). \tag{3}$$

Again in the Fig. 2 (b), the control loop supported by two communication links, the Plant-Controller link and the Controller-Plant link. Here, the Controller-Plant link is perfect with no time delay[2]. The Plant-Controller link is not perfect and with random time delays which follow exponential distribution.

A first-order linear feedback control system with a simple proportional controller is considered, where $G(s)=1/(s+a_0)$ and $C(s)=K$. Here, a_0 and K are fixed control parameters, which are known to the system.

B. AOI Evolution in Feedback Control Systems

We adopt the same network model as [7][8], where the network serve only one packet at a time. In other words, the freshest sample can be transmitted only when the previous packet has been delivered at the controller. The time delay in delivering one packet is random variable which is called serving time. Before taking a sample and transmitting a packet, the sensor need to wait for a certain a time, denoted as Z. As a result, the evolution of AOI is determined by the random delays $Y_0, Y_1, ...Y_n$ and the waiting time $Z_0, Z_1, ..., Z_n$.

Here, we adopt the evolution model as [7], which is shown in Fig. (3). Here, the sensor generates and submits a new update at the time slot instant S_0, S_1, ..., S_n. After packet i is delivered at time D_i, the sensor will introduce a waiting time Z_i. Thus, the actual AOI grows from $Y_{i-1}+Z_{i-1}$ to $Y_{i-1}+Z_{i-1}+Y_i$. This is how the AOI changes with time.

C. Control Performance and AOI

We use tracking error $e(t)$ to describe the control performance, where $e(t)$ is defined as the difference between the plant's output $y(t)$ and the reference $r(t)$ [10], i.e.,

$$e(t)=r(t)-y(t). \tag{4}$$

By combining the Equ. (3) and the Equ. (4), we can obtain the Laplace form of the tracking error as follows,

$$E(s)=R(s)-Y(s)=\frac{1}{1+C(s)G(s)e^{-\Delta(t)s}}R(s). \tag{5}$$

In Equ. (5), the *characteristic equation* of the control system is

$$1+C(s)G(s)e^{-\Delta(t)s}=0. \tag{6}$$

The roots of the characteristic equation is called poles, denoted as $X=\{x_1,x_2,...,x_i\}$. In general, performance with a unit impulse response is always used to evaluate the control design, i.e.,

$$r(t)=\int_{t=0}^{\infty}\delta(t), \tag{7}$$

and $R(s)=1/s$. Then, with proper coefficients $\{A_1, A_2, ..., A_i\}$, Equ. (5) becomes

$$E(s)=\frac{A_1}{s-x_1}+\frac{A_2}{s-x_2}+...+\frac{A_1}{s-x_i}. \tag{8}$$

The tracking error $e(t)$ can be obtained by inverse Laplace transform as

$$e(t)=A_1e^{x_1t}+A_2e^{x_2t}+...+A_ie^{x_it}. \tag{9}$$

As a result, the control performance is mainly influenced by AOI $\Delta(t)$, because AOI determines the locations of the poles.

III. Problem Formulation

In this section, we first define AOSI to describe the relationship between AOI and control stability. Then, we formulate a packet management optimization problem by minimizing AOSI, where the system can achieve the maximum control performance.

[2]This assumption is widely used [1][10][13], and also is true for the scenario that the controller and the actuator are together.

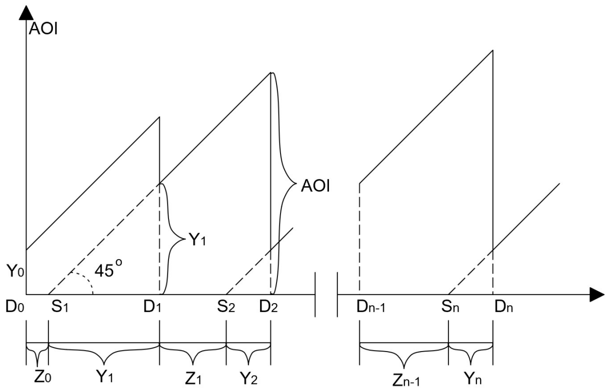

Fig. 3. A typical evolution of AOI. Z is the waiting time, and Y is the serving time.

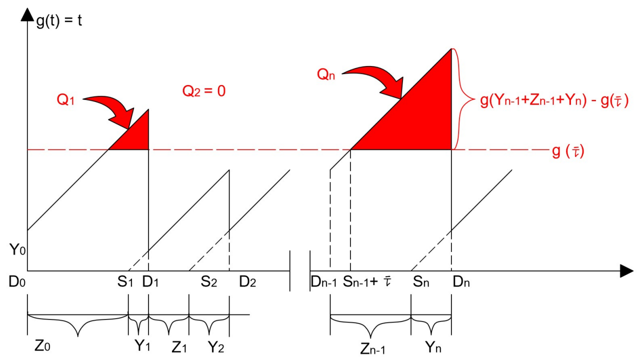

Fig. 4. By calculating the area of the red parts, we can obtain the average AOSI.

A. Stability and AOSI

It is known to all that the rightmost pole, denoted as x_r, plays a key role and is called the *dominant pole*. We have

$$e(t) \approx A_r e^{x_r t}. \tag{10}$$

In general, the location of x_r moves right as $\Delta(t)$ grows. When $\Delta(t)$ exceeds a stability margin, denoted as τ, the real part of x_r becomes positive. Then, the tracking error becomes divergence, and then the control system is unstable. This is what a real-time control tries to avoid. Thus, we propose a simple metric called the *age of stale information* (AOSI). AOSI links the instability to AOI, and is expressed as $\eta(t) = \max\{\Delta(t) - \tau, 0\}$. According to [14], we have

$$\tau = \frac{acos(-a_0/K)}{K^2 - a_0^2} \tag{11}$$

where $acos(\cdot)$ is the arc-cosine function. Compared with conventional AOI, AOSI helps the systems avoid the unstable case as much as possible.

B. Problem Formulation

For a control system with unbounded time delay, we need to design the waiting time Z to avoid $\Delta(t) > \tau$ as much as possible by minimizing AOSI. In the time interval $[T_{i-1}, T_i]$, total AOSI indicates the area under $\Delta(t)$ and above the stability margin τ, which is shown as the red parts in Fig. 4. Then, AOSI can be expressed as

$$\int_0^{T_n} max\{\Delta(t) - \tau, 0\}dt \triangleq \sum_{i=0}^{n-1} Q_i, \tag{12}$$

where

$$Q_i = \begin{cases} \int_{T_{i-1}}^{T_i} \{\Delta(t) - \tau\}dt, & \text{for} \quad \Delta(t) \geq \tau \\ 0. & \text{for} \quad \Delta(t) < \tau \end{cases} \tag{13}$$

Then, the optimization problem can be expressed as

$$\min_{\hbar} : g_a = \lim_{n\to\infty} \frac{\sum_{i=0}^{n-1} Q_i}{\sum_{i=0}^{n-1} (Z_i + Y_{i+1})}. \tag{14}$$

In the above optimization problem, $\sum_{i=0}^{n-1} Q_i$ indicates total instability of the control system, while $\sum_{i=0}^{n-1} (Z_i + Y_{i+1})$ is the total control time. Thus, it improves the control performance by minimizing the average instability.

IV. Solution

In this section, we firstly simplify the optimization problem in Equ. (14). Then, we solve it based on the standard water-filling method. As a result, a new packet update policy is obtained, called the $\alpha - wait$.

In [7][8], it proved that there exists a *stationary deterministic* policy for this kind of problem. A stationary deterministic policy means that a waiting time Z_i can be optimized only based on the value of Y_{i-1}, i.e., the optimization process is memoryless. This kind of optimization problem can be solved by the water-filling method, i.e., there exists a water level θ and $Z_i = \max\{\theta - Y_i, 0\}$. In this way, the original optimization problem can be simplified as

$$\min_{\theta} : g_a = \lim_{n\to\infty} \frac{\sum_{i=0}^{n-1} Q_i}{\sum_{i=0}^{n-1} (Z_i + Y_{i+1})}. \tag{15}$$

where

$$Z_i = \max\{\theta - Y_i, 0\}. \tag{16}$$

In the above problem, only one variable θ needs to be optimized. Thus, we try to optimize θ by calculating the expectation of AOSI, which is shown in next subsections.

1) The Expectation of Q_i: To optimize θ, we calculate the value of Q_i in three cases, which is shown in Fig. 5.

Case 1 ($Y_{i-1} < \theta$) In this case, we have $Y_{i-1} + Z_{i-1} = \theta$. Considering that Y_i is an exponentially distributed variable

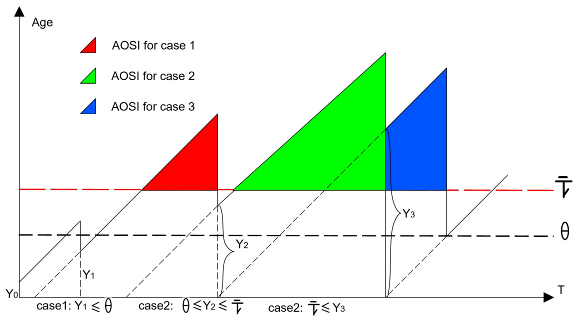

Fig. 5. Q_i can be calculated in 3 cases.

with the parameter λ, the expectation of Q_i, in this case, can be calculated as

$$\mathbb{E}\{Q_1\} = \int\limits_{\tau-\theta}^{\infty} (t-(\tau-\theta))f(t)dt, \tag{17}$$

where $f(t)$ is the *probability density function* (PDF) of Y_i, i.e.,

$$f(t) = \frac{1}{\lambda} e^{-\frac{t}{\lambda}}. \tag{18}$$

Then, we have

$$\mathbb{E}\{Q_1\} = \lambda e^{\frac{-\tau+\theta}{\lambda}}. \tag{19}$$

Case 2 ($\theta \leq Y_{i-1} < \tau$) In this case, we have $Z_{i-1} = 0$. Then, the expectation of Q_i, in this case, can be calculated as

$$\begin{aligned}\mathbb{E}\{Q_2\} &= \int\limits_{\tau-y_{i-1}}^{\infty} (t-(\tau-y_{i-1}))f(t)dt \\ &= \lambda e^{\frac{-\tau+y_{i-1}}{\lambda}}.\end{aligned} \tag{20}$$

Case 3 ($Y_{i-1} \geq \tau$) In this case, we still have $Z_{i-1} = 0$. Then, the expectation of Q_i, in this case, can be calculated as

$$\begin{aligned}\mathbb{E}\{Q_3\} &= \int\limits_{0}^{\infty} (t+y_{i-1}-\tau)f(t)dt \\ &= \lambda - \tau + y_{i-1}.\end{aligned} \tag{21}$$

2) The Expectation of AOSI: Consider that y_{i-1} is also an exponentially distributed variable with the parameter λ. Then, the average Q_i can be calculated as

$$\begin{aligned}\mathbb{E}\{Q_i\} &= \underbrace{\lambda e^{\frac{-\tau+\theta}{\lambda}}(1-e^{-\frac{\theta}{\lambda}})}_{Case1} + \underbrace{\int\limits_{\theta}^{\tau} Q_2 f(y)dy}_{Case2} + \underbrace{\int\limits_{\tau}^{\infty} Q_3 f(y)dy}_{Case3} \\ &= (\tau-\theta+\lambda+\lambda e^{\frac{\theta}{\lambda}})e^{-\frac{\tau}{\lambda}}.\end{aligned} \tag{22}$$

Similarly, the expectation of $T_i - T_{i-1}$ can be obtained

$$\begin{aligned}\mathbb{E}\{T_i - T_{i-1}\} &= \underbrace{\int\limits_{0}^{\theta} (\theta-y)f(y)dy}_{Case1} + \underbrace{\lambda}_{Case2+Case3} \\ &= \theta + \lambda e^{-\frac{\theta}{\lambda}}.\end{aligned} \tag{23}$$

Then, the average AOSI $g_a(\theta)$ is

$$\begin{aligned}g_a(\theta) &= \frac{\mathbb{E}\{Q_i\}}{\mathbb{E}\{T_i - T_{i-1}\}} \\ &= \frac{\tau-\theta+\lambda+\lambda e^{\frac{\theta}{\lambda}}}{\theta+\lambda e^{-\frac{\theta}{\lambda}}} e^{-\frac{\tau}{\lambda}}\end{aligned} \tag{24}$$

3) Optimal θ^*: We can obtain $\frac{\partial g(\theta)}{\partial \theta}$ as

$$\frac{\partial g(\theta)}{\partial \theta} = \frac{e^{\frac{-\tau+\theta}{\lambda}}\left(-1+e^{\frac{\theta}{\lambda}}\right)\varphi(\theta)}{(\lambda+\theta e^{\frac{\theta}{\lambda}})^2}, \tag{25}$$

where

$$\varphi(\theta) = -(\tau + \lambda e^{\frac{\theta}{\lambda}} - \theta(1+e^{\frac{\theta}{\lambda}})). \tag{26}$$

Here, it's not difficult to verify

$$\frac{e^{\frac{-\tau+\theta}{\lambda}}\left(-1+e^{\frac{\theta}{\lambda}}\right)}{(\lambda+\theta e^{\frac{\theta}{\lambda}})^2} > 0, \tag{27}$$

and

$$\frac{\partial \varphi(\theta)}{\partial \theta} = 1 + \frac{\theta}{\lambda} e^{\frac{\theta}{\lambda}} > 0. \tag{28}$$

Additionally, $\varphi(\theta=0) = -(\tau+\theta) < 0$ and $\varphi(\theta=\infty) > 0$. The above results indicate that the average AOSI is a U-shape curve as the water threshold θ grows. Then, the optimal θ^* is the solution to $\varphi(\theta) = 0$.

V. Simulation Results

A. Parameters Setting

A plant $G(s) = \frac{1}{s+a_0}$ with a negative proportional controller $C(s) = -K$ is considered in this section. To obtained MSE results, the total time of the entire control process is assumed as $T_n = 100$s, while the control sampling rate is 1KHz, i.e., the control interval T_s is 0.001s. The reference is assumed as the typical unit impulse response, i.e., $r(t) = \int\limits_{t=0}^{\infty} \delta(t)$. $Y_i \in (0, u)$ obeys to an exponential distribution with parameter λ, where the expectation of transmission time $\mathbb{E}\{Y_i\} = \frac{1}{\lambda} = \{0, 0.1, ..., 1.5\}$. The proportional parameter is $K = \{0.5, 1, ..., 5\}$. Additionally, the simulation results are obtained based on the Monte Carlo method by Matlab, where the number of Monte Carlo simulation is 1000.

For control performance, MSE and the final error is investigated in this section, where MSE is defined as

$$E_{mse} = \frac{1}{1+T_n/T_s} \sum_{t=\{0,T_s,2T_s,...,T_n\}} \{(y(t)-r(t))^2\}. \tag{29}$$

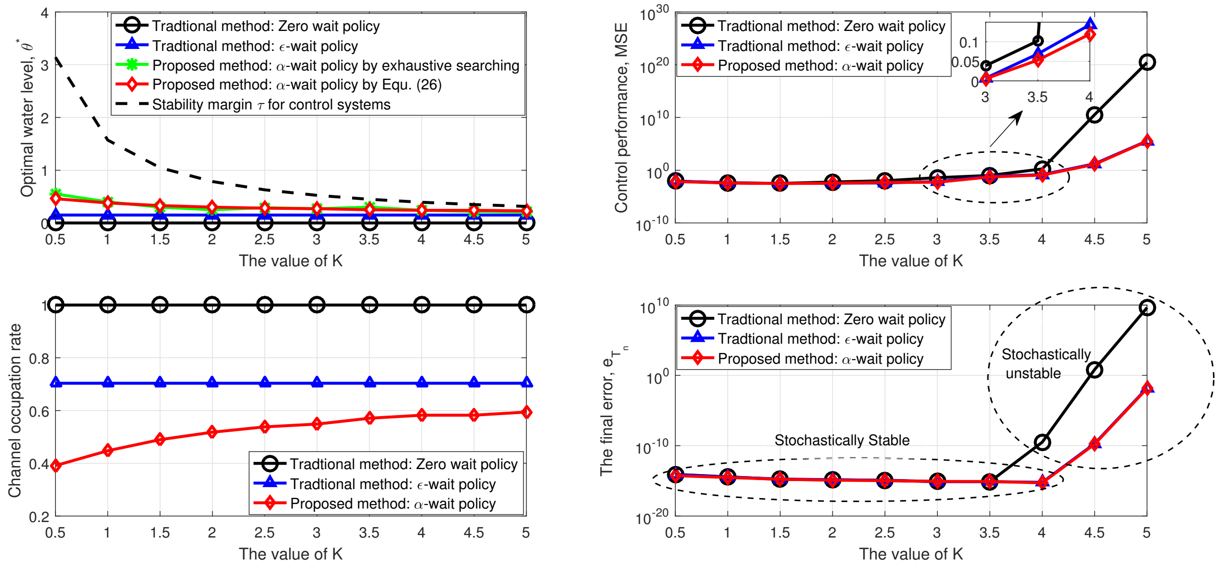

Fig. 6. Simulation results: Control performance and communication cost vers the value of the controller parameter K.

Additionally, it is necessary to guarantee that the final error e_{T_n} converges to 0, where e_{T_n} denotes the tracking error at the last time constant. If $e_{T_n} \to 0$, the control system is stochastically stable. Otherwise, it is stochastically unstable.

For communication cost, the channel occupation rate is introduced, where it is defined as

$$S = \frac{Y_i}{Y_i + Z_i}. \tag{30}$$

Then, minimizing the average channel occupation rate can reduce communication cost.

B. Numerical Results

As shown in the top left of Fig. 6, fixed water levels are obtained by the zero-wait policy and the minimizing AOI policy (blue and black curves) since the control property has not been taken into account in traditional methods. θ^* obtained by the proposed method decreases as K grows. The reason is that τ (see Equ. (11)) decreases with K, where we also have $\theta^* < \tau$. Additionally, results obtained by exhaustive searching (green curve) match the theoretical results obtained by Equ. (26) very well.

As shown in the lower left of Fig. 6, wireless communication resource consumption can be significantly reduced by the proposed method, e.g., the wireless consumption reduces almost half (0.72 to 0.39) compared to minimizing AOI when $K = 0.5$. As K grows, young-aged information becomes stale, i.e., AOSI is going to be equivalent to AOI.

As shown in both the top right and the lower right of Fig. 6, control performance obtained by the proposed method is very closed to the one obtained by minimizing AOI, while the zero-wait policy has the worst performance. The reason is that stochastic stability is essential for this system with unbounded time delays. When $K \leq 4$, both AOI and AOSI can guarantee the system is stochastically stable, while the latter one can also reduce the communication cost. When $K > 4$, the final error e_{T_n} does not converge to 0, i.e., all control performance is not acceptable.

VI. Conclusions

In this paper, we studied how to manage the communication packet in real-time feedback control systems. We found that minimizing AOI is not always equivalent to maximizing control performance. Thus, we defined AOSI to link AOI to the control stability. By minimizing AOSI, a new packet update method was designed, which could maximize the control performance, and also significantly reduce the communication cost.

References

[1] P. Park, S. Coleri Ergen, C. Fischione, C. Lu, and K. H. Johansson, "Wireless Network Design for Control Systems: A Survey," *IEEE Communications Surveys & Tutorials*, vol. 20, no. 2, pp. 978-1013, Secondquarter 2018.

[2] M. Li, Z. Li, and A. V. Vasilakos, "A Survey on Topology Control in Wireless Sensor Networks: Taxonomy, Comparative Study, and Open Issues," *Proceedings of the IEEE*, vol. 101, no. 12, pp. 2538-2557, Dec. 2013.

[3] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update? in *Proc. IEEE INFOCOM 2012*, Apr. 2012, pp. 2731C2735.

[4] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *Proc. IEEE INFOCOM*, Apr. 2012, pp. 2731C2735.

[5] R. Yates and S. Kaul, "Real-time status updating: Multiple sources," in *Proc. IEEE Int. Symp. Inform. Theory*, Jul. 2012

[6] R. Yates, "Lazy is timely: Status updates by an energy harvesting source, in *Proc. IEEE Int. Symp. Inform. Theory*, 2015.

[7] Y. Sun, E. Uysal-Biyikoglu, R. Yates, C. E. Koksal and N. B. Shroff, "Update or wait: How to keep your data fresh," in *proc. IEEE INFOCOM 2016*, San Francisco, CA, 2016, pp. 1-9.

[8] Y. Sun, E. Uysal-Biyikoglu, R. D. Yates, C. E. Koksal and N. B. Shroff, "Update or wait: How to keep your data fresh," in *IEEE Transactions on Information Theory*, vol. 63, no. 11, pp. 7492-7508, Nov. 2017.

[9] J. P. Champati, M. H. Mamduhi, K. H. Johansson, and J. Gross, "Performance characterization using AoI in a single-loop networked control system," in *pro. IEEE INFOCOM 2019,* Paris, France, 2019, pp. 197-203.

[10] R. C. Dorf and R. H. Bishop, "Modern control systems (13st Edition)", Boston, 2016, ISBN:978-0-13-440762-3.

[11] C. Y. Kao, and B. Lincoln, "Simple stability criteria for systems with time-varying delays," *Automatica*, vol.40, no. 8, pp. 1429-1434 , 2004.

[12] A. Kosta, N. Pappas, A. Ephremides, and V. Angelakis, "Age and value of information: Non-linear age case," in *Pro. 2017 IEEE International Symposium on Information Theory (ISIT)*, Aachen, 2017, pp. 326-330.

[13] A. Ahlen, J. Akerberg, M. Eriksson, et al, "Toward wireless control in industrial process automation: a case study at a paper Mill," *IEEE Control Systems Magazine*, vol.39, no.5, pp. 36-57, 2019.

[14] Rifat Sipahi, Silviu-iulian Niculesu, Chaouki T. Abdallah, Wim Michiels, and Keqin Gu, "Stability and Stabilization of Systems with Time Delay," *IEEE Control Systems Magazine*, vol.31, no.1, pp. 38-65, 2011.